Maximum *A Posteriori* Refractivity Estimation from Radar Clutter using a Markov Model for Microwave Propagation

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Abstract

This paper addresses the problem of estimating range-varying parameters of the height-dependent index of refraction over the sea surface in order to predict ducted microwave propagation loss. Refractivity estimation is performed using a Markov model for microwave radar clutter returns from the sea surface. Specifically, the parabolic approximation for numerical solution of the wave equation is used to formulate the problem within a non-linear recursive Bayesian state estimation framework. Solution for the maximum a posteriori (MAP) sequence of range-varying refractivity parameters, given log-amplitude clutter versus range data, is achieved using a technique based on the Viterbi algorithm. Simulation and real data results based on experiments performed off Wallops Island, Virginia are presented which quantify the technique's ability to predict propagation loss at 3 Ghz.

1. INTRODUCTION

The refractivity structure associated with the capping inversion of the marine atmospheric boundary layer often causes ducted microwave propagation [1], [2]. Synoptic monitoring of ducting conditions by direct measurement of the three-dimensional humidity and temperature profiles, which determine refractivity, is difficult and expensive [3]. Thus this paper addresses the problem of estimating refractivity from clutter (RFC). In previous work, simple global parameterizations of the range and height dependent refractivity profile have been fitted to clutter returns, producing some promising real data results in several instances [4]. However, in more complex range-varying scenarios, the number of global parameters required becomes too large to handle efficiently. In this paper, the parabolic approximation for numerical solution of the wave equation is used to formulate the more general range-varying refractivity estimation problem within a non-linear recursive Bayesian state estimation framework. The potential advantage of this state-space formulation of RFC is that it can be solved efficiently using sequential recursive Bayesian methods. This approach also imposes smoothness constraints on physically-realizable refractivity parameters. As with other RFC methods, the final objective is to predict propagation loss as a function of range and height which can be achieved by numerical solution of the wave equation using the estimated refractivity profile. Such propagation loss predictions are known as "coverage diagrams" and are often used as tactical decision aids to naval radar operators.

2. MODEL FORMULATION

Numerical solution for the electromagnetic field at range, x, and height, z, due to ducted propagation in inhomogeneous tropospheric conditions is commonly performed by using the parabolic equation (PE) approximation of the wave equation. In particular, the split-step Fourier PE solution [5] recursively computes the field, $u(x_{k+1}, z)$, at range, $x_{k+1} = x_k + \delta x$, as a function of height, z, given the solution at range, x, using a linear transformation given by:

$$u(x_{k+1}, z) = \exp\left\{ j\frac{k}{2}\left(\eta^2 + \frac{2z}{a_e} - 1\right)\delta x \right\} \times \mathbf{F}^{-1}\left(\exp\left\{ -j\frac{p^2 \delta x}{2k} \right\} \mathbf{F}\left\{ u(x_k, z) \right\} \right) \tag{1}$$

which amounts to assuming Fresnel diffraction through a thin phase-screen at each range step. The height-dependent refractivity profile between x_k and $x_k + \delta x$, is denoted, η, which enters into the phase screen term, which is the first complex exponential in (1). Other symbols in (1) are the radius of the earth, a_e, and the spatial Fourier transform operator, F, taken with respect to height, z. Consider now the case where the refractivity profile, $\eta(z, x_k, \mathbf{g}_k)$, is modeled as being a non-linear function of an uncertain random parameter vector, $\mathbf{g}_k$, whose range dependence is Markovian, i.e.

$$\mathbf{g}_{k+1} = \mathbf{A}\mathbf{g}_k + \mathbf{w}_k \tag{2}$$

where the known transition matrix $\mathbf{A}$ constrains the smoothness of the parameter variation across small range steps and the independent random vectors, $\mathbf{w}_k$, model uncertain variations between ranges. Now defining the vector of complex field values over height, $\mathbf{u}_k = [u(z_1, x_k), \ldots u(z_N, x_k)]^T$, at range step k, equation (1) can be written as:

$$\mathbf{u}_{k+1} = \mathbf{f}(\mathbf{u}_k, \mathbf{g}_k) \tag{3}$$

where the vector-valued function $\mathbf{f}(\cdot,\cdot)$ represents the split-step Fourier solution for the field. Putting $\mathbf{u}_k$ and $\mathbf{g}_k$ of (2) and (3) into a single state vector, $\mathbf{x}_k = [\mathbf{g}_k, \mathbf{u}_k]^T$, the electromagnetic field and range-varying refractivity parameters are constrained by a non-linear set of equations given by:

$$\begin{bmatrix} \mathbf{g}_k \\ \mathbf{u}_k \end{bmatrix} = \begin{bmatrix} \mathbf{A}\mathbf{g}_{k-1} \\ \mathbf{f}\left(\mathbf{u}_{k-1}, \mathbf{g}_{k-1}\right) \end{bmatrix} + \begin{bmatrix} \mathbf{w}_k \\ \mathbf{0} \end{bmatrix} \tag{4}$$

In this paper, $\mathbf{w}_k$ is modeled as zero-mean, Gaussian with covariance matrix Σ_W. In the above formulation, the process noise is only used in the model for the refractivity variables. Propagation of a weak forward random scattered field could, in principle, be handled by also including an additive process noise component in the state equations for $\mathbf{u}_k$. In the current application, the initial condition, $\mathbf{u}_0$, can assumed to be known from the antenna pattern of the radar. Historical observations and possibly an *in situ* measurement of refractivity at the radar can be used to form a prior distribution on $\mathbf{g}_o$.

Clutter returns from the sea surface can be expressed in terms of $\mathbf{x}_k$ by letting the matched-filtered radar return of the n^{th} pulse at the k^{th} slant range, denoted by $f_n(k)$, be expressed as:

$$f_n(k) = a_n(k) L(\mathbf{x}_k) + v_n(k) \tag{5}$$

where $a_n(k)$ is a complex, zero-mean, white Gaussian process with variance, $\sigma_a^2(k)$, representing local surface backscatter and $L(\mathbf{x}_k)$ is the magnitude of the field calculated at a nominal sea surface height. The receiver noise, $v_n(k)$, is modeled here as additional zero-mean complex white Gaussian noise process with variance, σ_v^2. In effect, (5) models the clutter return as the propagation loss modulated by a random "speckle noise", whose variance is the backscatter cross-section of the sea surface, in additive noise. In the forward problem, the range-dependent refractivity parameter sequence, $\mathbf{g}_k$, could be used as input to a PE propagation model to compute the propagation loss. The goal here, however, is to estimate the sequence of refractivity parameters, $\mathbf{g}_k$, given an observation of microwave radar clutter return statistics.

A common statistic of the received data that is available in many radars is the pulse-position indicator (PPI) output, y_k. The PPI is typically formed in the radar by averaging N matched-filtered, log-amplitude pulses such that $y_k = \frac{20}{N}\sum_{n=1}^{N} \log\left| f_n(k) \right|$. For the model of (5), it can be shown [4] that the PPI output for large N, conditioned on $\mathbf{x}_k$, is approximately Gaussian distributed with mean $10\log\left[\sigma_a^2(k) L(\mathbf{x}_k) + \sigma_v^2\right] + 0.116$ and variance, which is a known constant, σ_y^2. Thus using (5) and noting that $L(\mathbf{x}_k) = \mathbf{e}_1^H \mathbf{x}_k \mathbf{x}_k^H \mathbf{e}_1$ where $\mathbf{e}_1 = [1, 0, \ldots, 0]^T$, the PPI clutter return can be modeled as:

$$y_k = \beta(\mathbf{x}_k) + \varepsilon_k \tag{6}$$

where $\beta(\mathbf{x}_k) = \frac{10}{\ln(10)} \ln\left(\mathbf{e}_1^H \mathbf{x}_k \mathbf{x}_k^H \mathbf{e}_1 \sigma_a(k) + \sigma_v^2\right) - const.$ and the ε_k are Gaussian random variables with constant variance, σ_y^2. Given the non-linear state-space formulation of (4) and (6), the objective is now to estimate the sequence of refractivity parameters, $\mathbf{g}_k$, given an observation of microwave radar returns, y_k.

3. RFC VIA PARTICLE FILTERING

A classical solution to the non-linear RFC state estimation problem would involve linearization of equations (4) and (6) and solution using the extended Kalman filter (EKF). Unfortunately, however, the appearance of η in the complex exponential of (1) makes the linearized model prone to instability. In this paper, therefore, the maximum a posteriori (MAP) estimate of range-dependent refractivity is computed using a Monte Carlo particle filter approximation to the Viterbi algorithm. The basic idea behind particle filtering is that the posterior distribution of the state sequence given the data can be represented by a set of random realizations (or "particles") instead of a continuous high-dimensional function. This approach was originally developed in Bayesian statistics literature [6, 7], but is beginning to receive attention in the signal processing literature [8,9, 10].

The particle filtering approach taken here follows the approach described in [10]. Suppose at range step, k, random realizations, $\mathbf{x}_{k-1}(i)$, $i = 1, \ldots, M$, are available from probability density, $p(x_{k-1} \mid y_1, \ldots, y_{k-1})$. Then realizations or particles, $x_k^*(i)$, from $p(\mathbf{x}_k \mid y_1,...,y_{k-1})$ can be obtained by using each of these particles as input to the state equation of (4) together with random samples $\mathbf{w}_k(i)$ drawn from the Normal distribution, $N(0,\Sigma_w)$. The PPI clutter measurement, y_k, at range bin, k, is then used to compute the Viterbi path weight by performing:

$$\begin{aligned} W_k(i) &= \log p(y_k \mid x_k(i)) + \\ &\max_j [W_{k-1}(j) + \log P(x_k(i) \mid x_{k-1}(j))] \end{aligned} \tag{7}$$

where $W_{k-1}(j) = \max_{x_1,...,x_{k-1}} (\log p(x_1,...,x_{k-1} \mid y_1,...,y_{k-1})$ and the transition probability distribution, $p(x_k(i) \mid x_{k-1}(j))$, is complex Gaussian with mean $x_{k-1}(j)$ and singular covariance matrix with $\sum_w$ in its upper left-hand block. The PPI clutter measurement, y_k, at range bin, k, can also be used to update the prior for the current range cell by evaluating the likelihood of each particle:

$$q_i = \frac{p(y_k \mid x_k^*(i))}{\sum_{j=1}^{N} p(y_k \mid x_k^*(i))} \tag{8}$$

which is a discrete approximation to the *a posteriori* probability density, $p(\mathbf{x}_k \mid y_1,...y_k)$, i.e. the probability mass at the sample points, $\mathbf{x}_k^*(i)$. Samples from $p(\mathbf{x}_k \mid y_1,...y_k)$ can now be approximated by bootstrap resampling *M* times from this discrete distribution such that $\Pr\{x_k(j) = x_k^*(i)\} = q_i$ [6]. For the RFC problem formulation:

$$p(y_k \mid \mathbf{x}_k) \propto \exp\left\{ -\frac{\left(y_k - \beta(\mathbf{x}_k)\right)^2}{2\sigma_\gamma^2} \right\} \tag{9}$$

since the log-amplitude PPI data is nearly Gaussian for large *N*. The MAP estimate of the refractivity parameter sequence at each range step is finding the argmax of $W_k(i)$ in (7) and then tracing back through the trellis.

The above procedure uses a data adaptive random grid approximation to the *a posteriori* density of the state. This importance sampling approach can become degenerate, i.e. have many redundant particles, for highly multi-modal $p(x_{k-1} \mid y_1,...,y_{k-1})$ which is the case in RFC. Degeneracy was overcome here by not resampling from (8) but rather simply propagating the particles via the state evolution of (4). This sacrifices asymptotic convergence for better performance with a limited number of particles and modest computational complexity.

4. RESULTS USING WALLOPS ISLAND DATA

To test the proposed refractivity estimation method, simulated PPI clutter data was generated based on real data from the SPANDAR radar at Wallops Island Virginia. The operating frequency was 2.85 GHz. with an antenna at a height of approximately 30 meters. The 4-element unknown parameter vector, $\mathbf{g}_k$, consisted of the heights and modified refractivity values, nominally at top and bottom of the trapping layer, in a standard tri-linear refractivity profile [cf. e.g. 4]. The $\mathbf{g}_k$ were range-varying over $\delta x = 1000$ m. increments according to (4). The simulated and real PPI clutter return, y_k, versus range with *N=128* snapshots are shown as the thickest solid lines in bottom panel of Figures 1 and 2, respectively. Ducting over the sea is responsible for the significant clutter observed at ranges beyond 20 km. The prior distribution for the height parameters was assumed uniformly distribution from 15 to 200 meters. The prior on the M-values constrained the total M-deficit to less than 65 M units and never more upward refracting than the standard. The backscatter cross-section is assumed to be constant and known over the entire range of interest in the simulation. The transition matrix, **A**, in (2) is the identity with process noise covariance chosen so as to be able to track duct height changes of a couple of meters per range step. In Figures 1 and 2, the true refractivity profiles are the solid black vertical traces in the top panels, respectively. Similarly, the MAP profile estimates are indicated by dashed vertical traces. The ground-truth and RFC-estimated coverage diagrams for simulated and real data are shown in the second and third panels of Figures 1 and 2, respectively. Note that RFC estimates closely track the ground-truth values in both simulation and with real data. The MAP RFC estimate was performed using *M=400* particles with a run-time of less than 30 minutes on a 500 MHz Pentium computer. More quantitative comparison of the accuracy of RFC is given in Table 1 where the mean absolute propagation loss error is given on a per trial basis for a set of 12 clutter maps. The average absolute error between coverage predictions made using RFC versus helicopter-based refractivity measurements compares favorably with those achieved using a single range-independent sounding made at the shore, midpoint, or 60 km. out at sea.

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Table 1: Propagation Loss Prediction Errors

	PPI (dB)		Transmission Loss (dB)			
Trial	*RFC*	*Helo*	*RFC*	*Shore*	*Midpoint*	*Sea*
9	5.19	6.93	5.82	3.50	5.39	5.53
10	6.24	7.94	4.14	3.50	5.39	5.53
11	6.13	6.87	7.94	3.50	5.39	5.53
12	4.91	8.50	5.19	2.81	3.60	4.94
13	5.95	9.02	6.36	2.81	3.60	4.94
14	6.80	9.22	4.54	7.68	7.01	8.23
15	6.05	9.31	5.89	7.64	6.97	8.19
18	5.91	8.23	7.26	8.00	7.80	8.98
19	5.55	7.21	10.59	8.00	7.80	8.98
22	6.82	6.76	8.61	5.47	6.87	5.91
23	9.87	8.97	7.85	5.47	6.87	5.91
24	5.90	6.93	5.68	5.47	6.87	5.91
Average	**6.28**	**7.99**	**6.66**	**5.32**	**6.13**	**6.55**

Figure 1: Simulated Refractivity, Coverage Diagrams, and Clutter for the Wallops Island Experiment

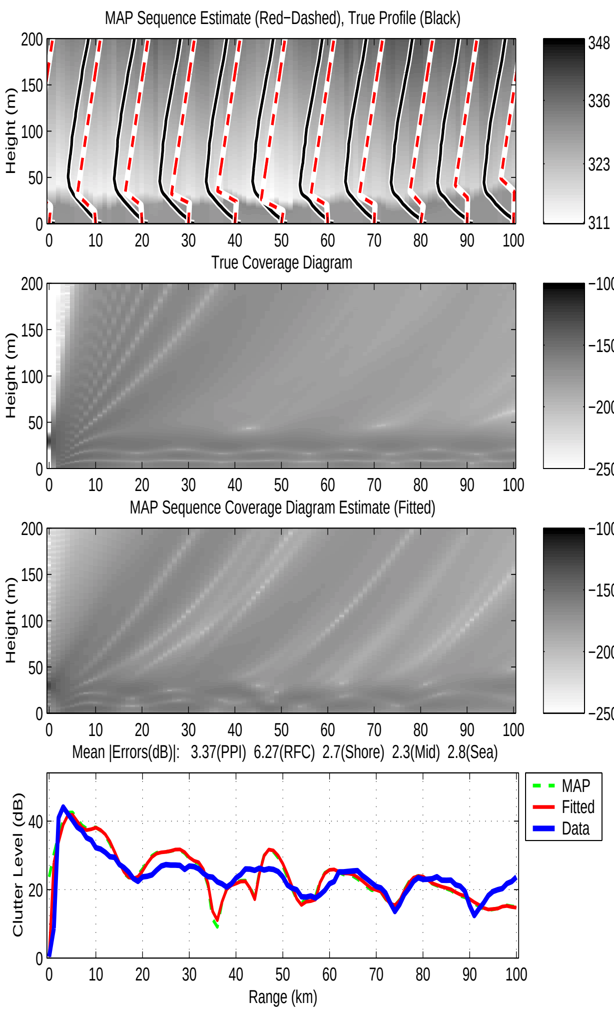
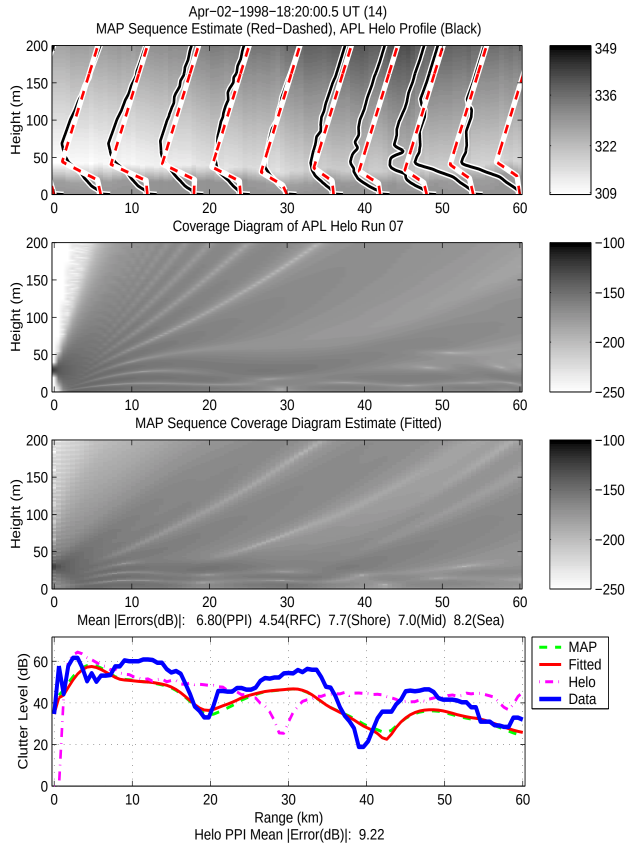


Figure 2: Real-Data Refractivity, Coverage Diagrams, and Clutter for the Wallops Island Experiment



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